Ducted Wind/Water Turbines and Propellers Revisited

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Introduction

There has been considerable effort and discussion in the literature (see, for example, Ref.s 1-8) concerning the potential for ducted wind/water turbines to outperform their unducted counterparts, i.e., surpass the Betz theoretical power extraction limit (see Ref.s 9 and 10 among others.) Results have been presented by Igar et al (Ref.s 1 & 2), Hansen et al (Ref. 4) and others that clearly, but empirically, demonstrate this tantalizing possibility. However, as discussed below, the majority of previous studies are based on an incomplete formulation of the problem that leads to incorrect limits for the performance benefits of ducted turbines. The simple but corrected formulation and results presented here: (a) provides a theoretical basis and verification of the potential available improvements from ducted flow configurations, (b) identifies a single, critical parameter, that controls that performance, (c) high-lights the appropriate non-dimensional scaling parameters and (d) provides a firm basis for further development of ducted wind/water turbine technology. Additionally, this formulation provides some interesting new results and insights for ducted/shrouded propeller propulsion.

Ducted Wind/Water Turbines

Figure 1 provides the geometry and nomenclature applied herein. All previous formulations for the unducted wind/water turbine and ducted propeller cases (see Ref.s 10 and 11, for example) have correctly imposed a pressure boundary condition at downstream infinity. To date, this has not been applied to ducted wind/water turbines (see Ref.'s 1-3 for example). Only Hansen et al (Ref. 4) used a closure condition far downstream in a multi-dimensional computational fluid dynamic (CFD) analysis of a simulated ducted wind turbine. In the current model the pressure boundary condition will be imposed at downstream infinity.

Referring to Figure 1, the governing equations are written for a control volume using a cut incorporating the turbine blades (modeled as an actuator-disc discontinuity with zero leakage around its edge) and the duct/shroud (with its attendant force on the flow), along with parallel, constant static pressure inflows and outflows at upstream and downstream infinity. With this, the conservation of mass, momentum and energy for a low speed and/or incompressible fluid leads to the following equation for the power extracted (note the equations are first presented in dimensional form and later in non-dimensional form per their power or propulsion application):

Power: $$P = \left[\frac{1}{2}\rho A_p\left(V_o^2 - V_a^2\right) + F_S\right]\left(V_o + V_a\right)/2 \qquad (1)$$

The axial shroud/duct force, F_S, in Equation 1 was modeled here as it has been for ducted propellers (as in Ref.11), i.e., F_S was taken to be directly related to the pressure jump across the

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disk/turbine through a non-dimensional shroud/duct force coefficient, C_s, which results in the expression:

Shroud Force: $$F_S = \tfrac{1}{2}\left[\rho A_p\left(V_o^2 - V_a^2\right)\right]C_S \tag{2}$$

Ref. 11 provides detail discussion on the relationship between this pressure jump and the circulation about the ring airfoil.

The resulting internal velocity at the turbine disk and thrust produced are than given as:

Velocity: $$V_p = \frac{1}{2}(1 + C_S)(V_o + V_a) \tag{3a}$$

Total Thrust: $$T = (1 + C_S)P / V_p = 2P/(V_o + V_a) \tag{3b}$$

Note that the unducted wind/water turbine case is recovered using C_S=0 and that these equations are a slight variant of those used for propeller propulsion (as presented by McCormick, Ref. 11 and others.) It is also noted that, and it shall be demonstrated below, the non-dimensional shroud/duct force coefficient, C_S, can be determined at any convenient level of power extraction, including zero, i.e., for the case of a clear duct.

From Equations 1-3 it is straightforward to show that the maximum power that can be extracted by a ducted wind/water turbine is given simply as:

$$C_{P\max} \equiv -P\Big/\left[\frac{1}{2}\rho A_p V_a^3\right] = \frac{16}{27}\left[1 + C_S\right] \tag{4}$$

which is shown in Figure 2 along with the attendant flow properties given as:

$$V_{oa_m} \equiv (V_o / V_a)_m = \frac{1}{3} \tag{5a}$$

$$V_{pa_m} \equiv (V_p / V_a)_m = A_{ip_m} \equiv (A_i / A_p)_m = \tfrac{2}{3}(1 + C_S) \tag{5b}$$

$$A_{op_m} \equiv (A_o / A_p)_m = (V_{pa} / V_{oa})_m = 2(1 + C_S) \tag{5c}$$

As shown in Equation 4 and Figure 2, this simple formulation captures the traditional bare/unducted wind turbine case (the Betz Power Limit of 16/27) at C_s=0 as but one of an infinite family of possibilities.

The validity and utility of the current formulation can best be demonstrated through comparison with the CFD results of Hansen et al (Ref. 4) where they presents a range of the power extraction levels for flow through an actuator disk simulating a pressure drop across a wind/water turbine.

Results were presented for the unducted case as well as an aerodynamically contoured ducted case with an aggressive exit area ratio, A_D/A_p=1.86. Figure 3 reproduces the results from Ref. 4 which were presented in terms of the thrust on the actuator disk defined as:

$$C_T \equiv T_p / \left(\tfrac{1}{2} \rho A_p V_a^2 \right) \tag{6}$$

where T_p was calculated from the pressure drop acting on the actuator disk area, A_p.

For the current formulation, applying Equations 1-3 provides the relationship:

$$C_P \equiv (1 + C_S / 2) C_T \left[1 + \sqrt{1 - C_T} \right] \tag{7}$$

which requires determination, by independent means, of the shroud/duct force coefficient C_S. This can be done by first noting that the current formulation applies for all power extraction levels including that of the clear duct case with zero power extraction. Conveniently, Hansen et al (Ref. 4) did provide the flow parameters for this case. In particular, they gave that V_{pa}= 1.83, which, when used in Equation 3a along with the fact that V_o=V_a for the clear duct, gives C_S=0.83.

The resulting comparison presented in Figure 3 shows that the simple one-dimensional inviscid flow model well represents the CFD results over the full range of the blade thrust for both the bare and ducted configuration. Not surprisingly, the CFD results produce a lower maximum power level for the ducted case due to the considerable viscous losses encountered for such an aggressive diffusion area ratio of 1.86.

To further relate the current formulation to earlier works (e.g., Ref.s 1-3); it is useful to first determine the pressure level at the exit plane, A_D, of Figure 1 using Equations 1-3 with Bernoulli's equation to write:

$$C_{p_D} \equiv \frac{p_D - p_a}{\frac{1}{2} \rho V_a^2} = V_{oa}^{\ 2} - \frac{V_{pa}^2}{A_{pD}^2} = V_{oa}^2 - \left(\frac{1 + C_s}{2} \right)^2 \left(1 - k + k A_{pD}^2 \right) \left(1 + V_{oa} \right)^2 \tag{8a}$$

where, as in Ref.s 1-4, the diffuser static pressure recovery efficiency coefficient, k, has been introduced to relate the pressure rise from the blade/disk location to the exit plane, and the area ratio is given in shorthand fashion as:

$$A_{pD} \equiv A_p / A_D \tag{8b}$$

Note the Equation 8a dictates that the exit pressure and diffusion levels cannot be employed as independent variables (as implied in earlier works, e.g. Ref.s 1-3) but rather must always satisfy this relation. Most importantly, in order to extract the maximum power possible, they must satisfy this relation with V_{oa} = 1/3, as dictated by Equation 5a.

With Equations 8a and 8b, it is also convenient to employ Igar's definition (Ref.s 1 & 2) of the maximum power available using Equations 1-6 to write that:

$$r \equiv \frac{27}{16} C_{P\max} = \frac{1}{2}\sqrt{(1-9C_{pD})/(1-k+kA_{pD}^{2})} \tag{9}$$

which differs significantly from that proposed by Igar (Ref. 2) and Riegler (Ref. 3) who did not impose the appropriate downstream pressure condition.

Results from use of Equation 9 are shown in Figure 4 for duct area diffusion ratios, A_{Dp} = 1.0 and 1.3 for a diffuser pressure recovery coefficient, k =1. Ducts with area diffusion ratios much larger than 1.3 are found to suffer significant losses (k<1 as was the case in Ref. 4) unless they are quite long, and thus too heavy to be practical. In fact, applying Equation 9 to Hansen et al's case (Ref. 4) that had k=0.83 with A_{Dp}=1.84, one can estimate that the viscous induced pressure losses in the diffuser reduced the maximum power extracted by nearly 20% below it's theoretical limit. Figure 4 clearly shows that ducted wind/water turbines are theoretically capable of extracting power levels significantly above those of their unducted counterparts for realistic levels of diffusion and exit pressure levels.

Ducted Propellers

As a final point, it is additionally noted that the formulation of Figure 1 also applies to the ducted propulsive propeller case with known power input. For this case Equations 1 and 2 can be rewritten as:

$$V_{oc}^{3} + V_{ac}V_{oc}^{2} + V_{ac}^{2}V_{oc} - 1 - V_{ac}^{3} = 0 \tag{10a}$$

where use has been made of the following definitions:

$$V_c \equiv \left(1/(1+C_S)\right)^{1/3} V_P \tag{10b}$$

$$V_P \equiv \left(4P/\rho A_p\right)^{1/3} \tag{10c}$$

(Note the "Power" velocity, V_P, of Equation 10c is closely related to the disk loading coefficient used by others, e.g., Ref. 11)

$$V_{oc} \equiv V_o/V_c \tag{10d}$$

$$V_{ac} \equiv V_a/V_c \tag{10e}$$

$$C_{SP} \equiv F_S / \frac{1}{2}\rho A_p V_P^2 \tag{10f}$$

The solution to the cubic Equation 10a can be approximated using a Taylor series to represent the explicit but more complex closed form exact solution as:

$$V_{oc} \approx 1 - \frac{1}{3} V_{ac} + \frac{4}{9} V_{ac} \tag{11}$$

This in turn can be used in Equation 3b to calculate the ducted systems total thrust in terms of a thrust coefficient defined as:

$$C_{TP} \equiv \frac{T}{\frac{1}{2} \rho A_p V_P^2} = \left(1 + C_S\right)^{1/3} / \left(V_{ac} + V_{oc}\right) \approx \left(1 + C_S\right)^{1/3} / \left(1 + \frac{2}{3} V_{ac} + \frac{4}{9} V_{ac}^2\right) \tag{12}$$

As shown in Figure 5, the unducted case for static flight ($C_{TP} = 1$) is recovered at $C_{SP}=0$ as but one of a family of cases, all of which are well represented by the simple polynomial approximation of Equation 12 for both static and forward flight conditions. This formulation can be further simplified by first noting that for the static case, $V_a=0$ and the duct exit plane pressure coefficient of Equation 8a with V_P replacing V_a can be used with the approximate form of Equation 12 and further Taylor series approximations to write that:

$$C_{TP0} \equiv \left(C_{TP}\right)_{V_a=0} = \left(1 + C_S\right)^{1/3} \approx \left(2A_{Dp}\right)^{1/3} - A_{Dp} C_{pD_P} \tag{13}$$

where, for current applications, it has been further assumed that the diffuser efficiency, k, is unity. The resulting Figure 6 shows that for the static case: (a) thrust increase of nearly 80% above the bare propeller level are attainable with moderate diffusion and exit plane suction pressures and (b) the handy approximation of Equation 13 gives a good representation over the regimes of interest.

Finally, combining Equations 12 & 13 leads to a simple relationship for forward flight effects on the thrust as

$$T/T_O = C_{TP} / C_{TPo} \approx 1 / \left(1 + \frac{2}{3} C_{TPo} V_{aP} + \frac{4}{9} C_{TPo}^2 V_{aP}^2\right) \tag{14a}$$

which is shown in Figure 7 to yield a very accurate approximation to the exact solution for virtually all values of forward velocity. The independent variable in Equation 14a can also be written as a generalization of that given by McCormick (Ref 11) and others as:

$$C_{TPo} V_{aP} = V_a / \sqrt{T_0 / 2(1 + C_S) \rho A_p} \tag{14a}$$

Concluding Remarks

As simple as the above formulation is, it is hard to overstate its importance or utility for ducted wind/water turbines. From its analytical predictions presented herein, it is observed that:

(a) ducted turbines are theoretically capable of extracting power significantly in excess of a bare wind/water turbine, (b) there is but a single parameter, the duct/shroud non-dimensional force coefficient, C_s, that determines the maximum power extractable, (c) for the first time the here-to-fore missing Betz-like core element has become available for use in the detailed design of the wind/water turbine blades' cross sectional shape along their spans so as to guarantee the capture of the maximum power available from the flow passing over the blade (Ref. 10 provides an excellent explanation of this approach for un-ducted wind turbines.) The explicit relationships presented here couple the design of the blades with their surrounding duct in a manner that must be satisfied in order to achieve optimal power extraction. With this new model in hand, a rational approach to the design of ducted wind/water turbines can precede with the potential for achieving the maximum power output available. Without it, all previous such designs must necessarily be considered potentially sub-optimal. It is also worthy of note that with the simple formulation presented here, one can straight forwardly predict the ducted wind/water turbine performance for all power levels based on the flow characteristics of the clear duct configuration.

Additionally, for ducted propellers, the current formulation produced a series of simple algebraic relations for predicting performance, correlating data and/or guiding preliminary design efforts with or without forward flight effects.

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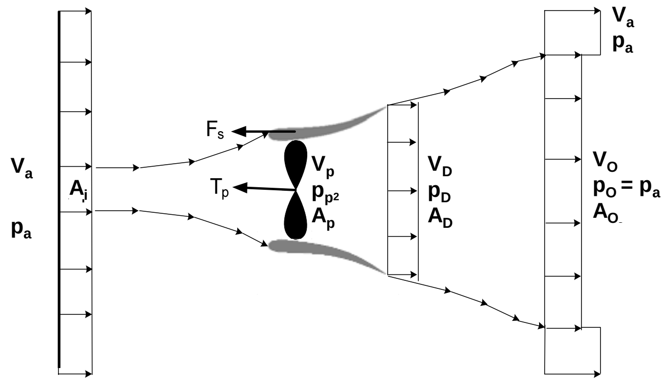

Figure 1: Ducted System Nomenclature

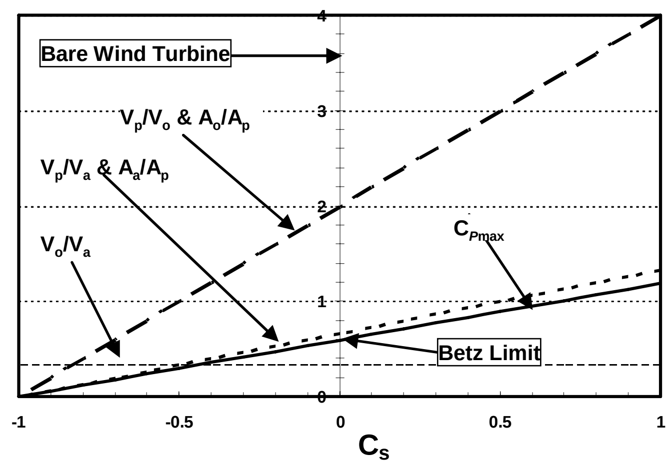

Figure 2: Wind Turbine Max Power Characteristics

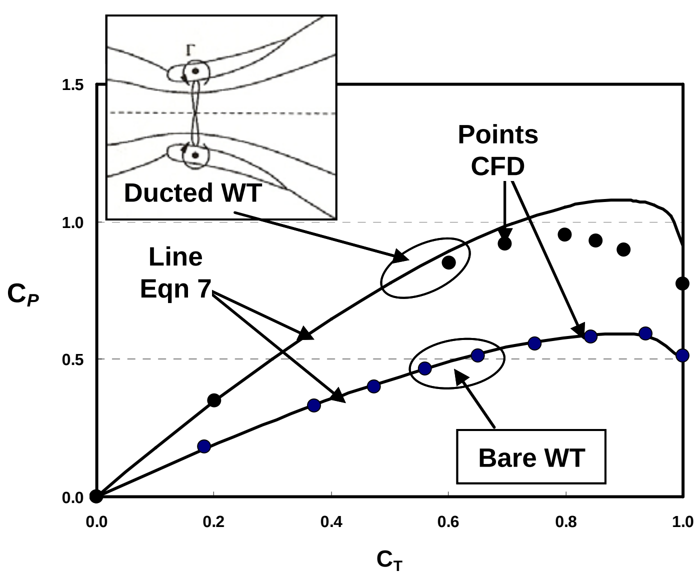

Figure 3: Comparison with CFD Analysis

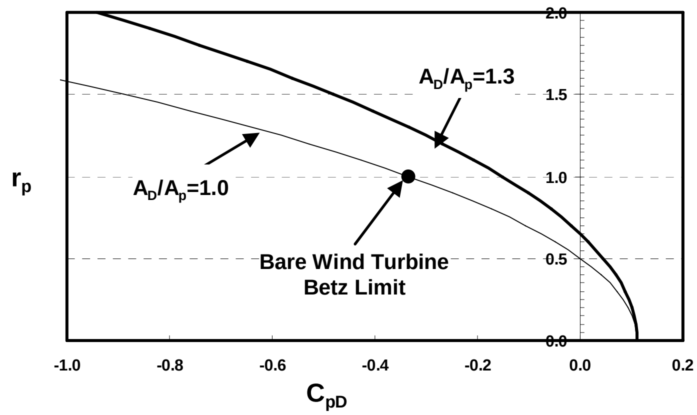

Figure 4: Wind Turbine Max Power Limits

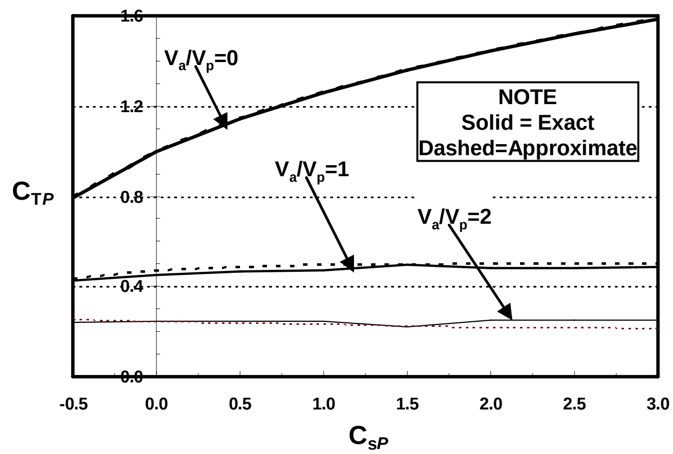

Figure 5: Ducted Prop Performance

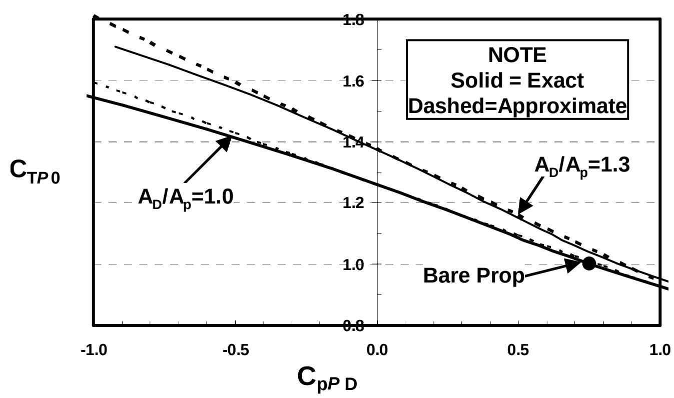

Figure 6: Static Ducted Prop Performance

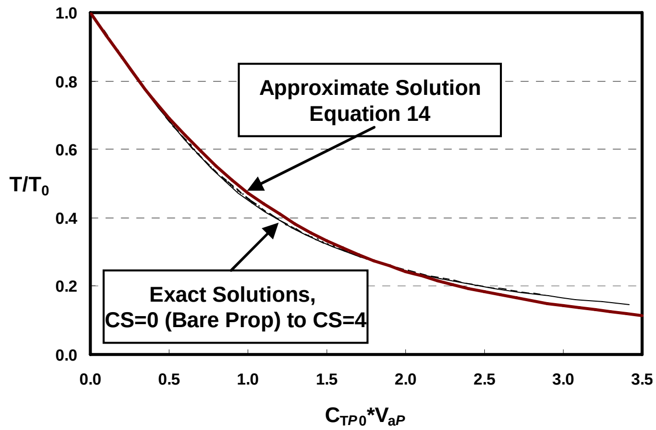

Figure 7: Ducted Prop Forward Velocity Effect